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APPLICATION OF THE THEORY OF LINEAR OPERATORS TO THE WAVEGUIDE DISCONTINUITY PROBLEMS

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ABSTRACT

A new technique, based on the matrix representation of linear operators, is presented to solve the inverse problem of finding out a linear operator of known eigenvalues and eigenvectors. The technique solves the discontinuity of a junction between a rectangular waveguide and and a shielded microstrip. Experimental measurements achieved are in good agreement with the theoretical results.

BASIC FORMULATION

The junction to be studied is shown in Fig. 1. It is composed of a transition from an empty rectangular waveguide to a shielded microstrip of the same width but with a smaller height. A cross-section at the discontinuity plane (z=0) is shown in Fig.2. As can be seen, the structure can be divided into three regions:

a- Region(1): $(0\leq x\leq a,\ \ 0\leq y\leq b\ ,\ z\leq 0)$,
b- Region(2): $(0\leq x\leq a,\ \ d_1\leq y\leq d_2,\ z\geq 0)$,
c- Region(3): $(0\leq x\leq a,\ \ d_2\leq y\leq b\ ,\ z\geq 0)$,

where regions (2) and (3) constitute together the shielded microstrip and region (1) constitutes the empty rectangular waveguide. All metallic boundaries are assumed to be perfectly conducting. The boundary conditions at the junction interface are obtained by matching the electromagnetic field components at both sides of the interface plane (z=0).

At the left hand side of the interface $(z=0^-)$, there exists only region(1) which is bounded by $(0<x<a,\ \ 0<y<b)$. The field components at this side can be expressed as:

$$E_x = \sum_{p=0}^{\infty}\sum_{q=1}^{\infty} E^x_{pq}\cos(p\pi x/a)\sin(q\pi y/b) \tag{1-a}$$

$$E_y = \sum_{p=1}^{\infty}\sum_{q=0}^{\infty} E^y_{pq}\sin(p\pi x/a)\cos(q\pi y/b) \tag{1-b}$$

$$E_z = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} E^z_{pq}\sin(p\pi x/a)\sin(q\pi y/b) \tag{1-c}$$

$$H_x = \sum_{p=1}^{\infty}\sum_{q=0}^{\infty} H^x_{pq}\sin(p\pi x/a)\cos(q\pi y/b) \tag{1-d}$$

$$H_y = \sum_{p=0}^{\infty}\sum_{q=1}^{\infty} H^y_{pq}\cos(p\pi x/a)\sin(q\pi y/b) \tag{1-e}$$

$$H_z = \sum_{\substack{p=0\\(p,q)\neq(0,0)}}^{\infty}\sum_{q=1}^{\infty} H^z_{pq}\cos(p\pi x/a)\cos(q\pi y/b) \tag{1-f}$$

where E^x_{pq} , E^y_{pq}, E^z_{pq}, H^x_{pq}, H^y_{pq} and H^z_{pq} are field expansion coefficients. If it is assumed that the wave is incident from $(z=-\infty)$, then incident and reflected modes are existing in region (1) and therefore any of the above field expansion coefficients is composed of both incident and reflected parts, e.g.:

$$E^x_{pq} = E^+_{pq} + E^-_{pq} \tag{2-a}$$

$$H^x_{pq} = H^+_{pq} - H^-_{pq} \tag{2-b}$$

The other expansion coefficients are linear combinations of E^+_{pq}, E^-_{pq} , H^+_{pq} and H^-_{pq} . The z-dependence of the (p,q) mode is assumed to have the form $\exp(-\gamma_{pq}z)$, where:

$$\gamma^2_{pq} = (p\pi/a)^2+(q\pi/b)^2-k_0^2,\ k_0^2 = \omega^2\mu_0\varepsilon_0 \tag{3}$$

The first region at the right hand side of the interface $(z=0^+)$ is a perfectly conducting step bounded by $(0\leq x\leq a\ ,\ 0\leq y\leq d_1)$. In this region there exists an unknown surface current $\underline{J}_s\ (x,y)$ given by:

$$\underline{J}_s\ (x,y) = J^x_s(x,y)\,\hat{\underline{i}} + J^y_s(x,y)\,\hat{\underline{j}} \tag{4}$$

The two components of this surface current can be expanded as follows:

$$J^x_s(x,y) = \sum_{p=0}^{\infty} J^x_{sp}(y)\cos(p\pi x/a) \tag{5-a}$$

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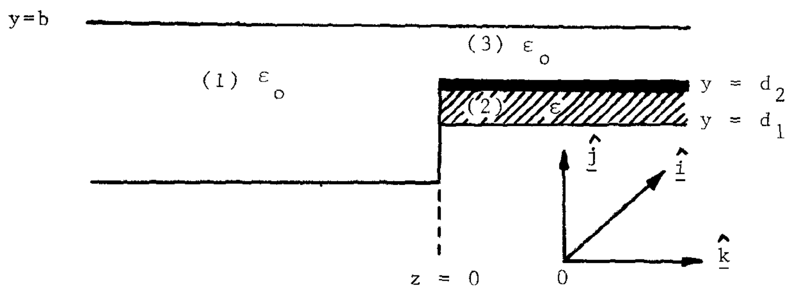

Fig.1 The junction between a waveguide & a microstrip

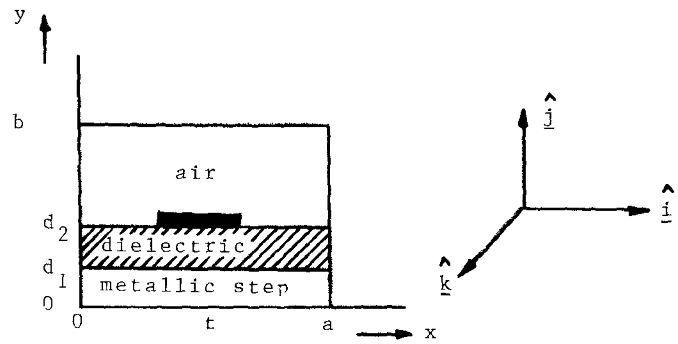

Fig. 2 A cross-section at the discontinuity plane

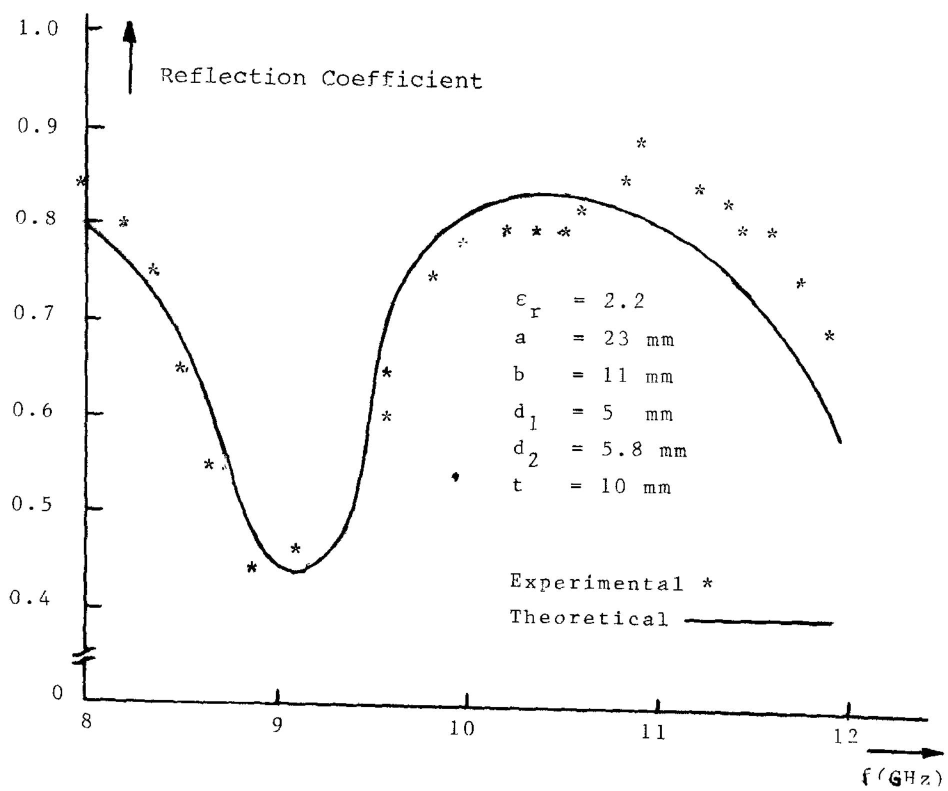

Fig.3 Reflection Coefficient of the dominant mode.

$$J_s^y(x,y) = \sum_{p=1}^{\infty} J_{sp}^y(y)\sin(p\pi x/a) \qquad (5\text{-b})$$

The second and third regions at right hand side of the interface are the two regions of the shielded microstrip that are bounded by $(0 \leq x \leq a,\ d_1 \leq y \leq d_2)$ and $(0 \leq x \leq a,\ d_2 \leq y \leq b)$. The field components in region (2) can be expressed as:

$$E_x = \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} T_n E_{np}^{x1} \cos(p\pi x/a) \sin\beta_{np}(y-d_1) \qquad (6\text{-a})$$

$$E_y = \sum_{p=1}^{\infty} \sum_{n=1}^{\infty} T_n E_{np}^{y1} \sin(p\pi x/a) \cos\beta_{np}(y-d_1) \qquad (6\text{-b})$$

$$E_z = \sum_{p=1}^{\infty} \sum_{n=1}^{\infty} T_n E_{np}^{z1} \sin(p\pi/x) \sin\beta_{np}(y-d_1) \qquad (6\text{-c})$$

$$H_x = \sum_{p=1}^{\infty} \sum_{n=1}^{\infty} T_n H_{np}^{x1} \sin(p\pi x/a) \cos\beta_{np}(y-d_1) \qquad (6\text{-d})$$

$$H_y = \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} T_n H_{np}^{y1} \cos(p\pi x/a) \sin\beta_{np}(y-d_1) \qquad (6\text{-e})$$

$$H_z = \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} T_n H_{np}^{z1} \cos(p\pi x/a) \cos\beta_{np}(y-d_1) \qquad (6\text{-f})$$

where the z-dependence of the n-th normal mode in the shielded microstrip is assumed to have the form $\exp(-\gamma_n z)$, and:

$$\beta_{np}^2 = \varkappa k_0^2 + \gamma_n^2 - (p\pi/a)^2 \qquad (7)$$

E_{np}^{x1}, E_{np}^{y1}, E_{np}^{z1}, H_{np}^{x1}, H_{np}^{y1} and H_{np}^{z1} are expansion coefficients.

The field components in region (3) are given by a set of equations similar to equations (6) when replacing $E_{np}^{x1}, \dots$ and H_{np}^{z1} by $E_{np}^{x2}, \dots$ and H_{np}^{z2}, respectively, and $\beta_{np}(y-d_1)$ by $\alpha_{np}(b-y)$, where:

$$\alpha_{np}^2 = k_0^2 + \gamma_n^2 - (p\pi/a)^2 \qquad (8)$$

The expansion coefficients $E_{np}^{x1}, \dots H_{np}^{z1}$, $E_{np}^{x2}, \dots, H_{np}^{z2}$ of the n-th normal mode are all related together, and if each mode is normalized (e.g. to carry a unity power), then the only unknowns in equations (6) and the corresponding equations representing the field in region (3) are the transmission coefficients of the different microstrip modes T_n.

By matching the field components at the left and right hand sides of the interface plane (z=0), the following equations are obtained:

$$g_p^e(y) + \int_{d2}^{b} f_p^e(y')K_p^e(y,y')\,dy' = h_p^e(y) - d_p^e(y),$$

$(d_2 \leq y \leq b)$ and $p = 0, 1, 2, \dots$ (9-a)

$$f_p^h(y) + \int_{d2}^{b} g_p^h(y')k_p^h(y,y')\,dy' = h_p^h(y) - d_p^h(y),$$

$(d_2 \leq y \leq b)$ and $p = 1, 2, 3, \dots$ (9-b)

Let the linear integral operators $\bar{L}_p^e$ and $\bar{L}_p^h$ be defined as:

$$\bar{L}_p^e\,[f(y)] = \int_{d2}^{b} f(y')k_p^e(y,y')\,dy',$$

$(d_2 \leq y \leq b)$ and $p = 0, 1, 2, \dots$ (10-a)

$$\bar{L}_p^h\,[g(y)] = \int_{d2}^{b} g(y')K_p^h(y,y')\,dy',$$

$(d_2 \leq y \leq b)$ and $p = 1, 2, 3, \dots$ (10-b)

Also, let the linear operators L_p^e and L_p^h be defined in such a way that their eigenfunctions are $\sin\alpha_{np}(b-y)$ and $\cos\alpha_{np}(b-y)$, respectively, with corresponding eigenvalues (γ_n/α_{np}^2) and $(1/\gamma_n)$, respectively. Equation (9) can be then re-writen as:

$$\bar{L}_p^e\,[f_p^e(y)] + \bar{L}_p^e\,[f_p^e(y)] = h_p^e(y) - d_p^e(y)$$

$(d_2 \leq y \leq b)$ and $p = 0, 1, 2, \dots$ (11-a)

$$L_p^h\,[f_p^h(y)] + \bar{L}_p^h\,[f_p^h(y)] = h_p^h(y) - d_p^h(y)$$

$(d_2 \leq y \leq b)$ and $p = 1, 2, 3, \dots$ (11-b)

MATRIX REPRESENTATION OF OPERATOR EQUATIONS

According to the concept of matrix representations of linear operators [1-4] the following can be stated:
The matrix representation "A" of the linear operator "L" with respect to the set of complete and orthogonal functions $\{u_n\}$ has the elements a_{nm} given by:

$$a_{nm} = \langle u_n^*, L[u_m]\rangle \qquad (12)$$

where the form $\langle\phi,\psi\rangle$ represents the dot product of the two functions ϕ and ψ. Another matrix representation of the same operator is the diagonal matrix "Λ" which represents "L" with respect to the set of its eigenvectors $\{v_n\}$. The elements of "Λ" are just the eigenvalues of "L". The two matrices "A" and "Λ" are related by:

$$A = T \Lambda T^{-1} \quad (13)$$

where "T" is the linear transformation that transforms the representation with respect to the set $\{v_n\}$ to that with respect to the set $\{u_n\}$. The elements t_{nm} of the matrix "T" are given by:

$$t_{nm} = \langle u_n^* , v_m \rangle \quad (14)$$

Any function "f" belonging to the domain of the linear operator "L" has also two column vector representations "X" and "$\bar{X}$" with respect to the sets $\{u_n\}$ and $\{v_n\}$, respectively. The two vectors are related by:

$$X = T \bar{X} \quad (15)$$

The elements x_n of the column vector "X" are given by:

$$x_n = \langle u_n^* , f \rangle \quad (16)$$

Because the period of definition of all the functions existing in equation (11) as $(d_2 \leq y \leq b)$, the dot product of the two functions f(y) and g(y), defined on $[d_2 , b]$, may be defined as:

$$\langle f , g \rangle = \int_{d2}^{b} f(y)\, g(y)\, dy \quad (17)$$

The set of functions $\{u_n(y)\}$ defined by:

$$u_n(y) = \exp(j2n\pi y/(b-d_2))/ \sqrt{(b-d_2)},$$

$$n = 0 , \pm 1, \pm 2, \ldots \quad (18)$$

is a complete and orthogonal one on $[d_2 , b]$.

The operator equation (11) can be easily proved to have the following matrix forms:

$$(T_p^e \Lambda_p^e + A_p^{-e} T_p^e)\, X_p^{-e} = Y_p^e ,$$

$$p = 0, 1, 2, \ldots \quad (19\text{-a})$$

$$(T_p^h \Lambda_p^h + A_p^{-h} T_p^h)\, X_p^h = Y_p^h ,$$

$$p = 1, 2, 3, \ldots \quad (19\text{-b})$$

NUMERICAL RESULTS

If the amplitude distribution of the incident modes E_{pq}^+ and H_{pq}^+ is known equation (19) can be solved numerically for the amplitude distribution of the transmitted modes T_n. The amplitude distribution of the reflected modes E_{pq}^- and H_{pq}^- is obtained then in terms of the incident and transmitted distributions.

The simple case of dominant mode incidence, in which all coefficients E_{pq}^+ and H_{pq}^+ are zeros except for H_{10}^+, is solved. The theoretical and experimental values of waveguide dominant mode reflection coefficient defined as:

$$R_{10}^h = (H_{10}^- / H_{10}^+) \quad (20)$$

are plotted in Fig. 3 over the X-band for the sake of comparison. The results are in good agreement except for the higher range of the X-band. The following reasons account for the deviation between the two plots.

a- The accuracy in reading the experimental values, as they were taken visually from the screen of the network analyzer.

b- Tolerance in the measured structure dimensions and the value of the substrate dielectric constant.

c- The truncation made in the matrix equation (19) which is essentially of infinite order.

d- The parasitic discontinuities existing in the measured structure due to the surface finishing as well as the solder points.

The marked deviation at high frequencies between the theoretical and experimental results is mainly due to the fact that the different modes of the shielded microstrip were obtained numerically by looking for the roots of a determinental equation. This contains successive poles and zeros that become nearer as the frequency is increased, accordingly the accuracy of finding the roots is decreased and the possibility of missing a root is increased as the frequency is increased.

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